Adaptive Testing by Bayesian Networks with Application to Language Assessment

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Abstract. We present a general procedure for computerized adaptive testing based on probabilistic graphical models, and show on a real-world benchmark how this procedure can increase the internal consistency of the test and reduce the number of questions without affecting accuracy.

Keywords: Computerized adaptive testing; Bayesian networks; Entropy.

1 Introduction

The goal of Computer Adaptive Testing (CAT) is to reduce the assessment time and to challenge test takers by adapting the sequence of questions to their ability level. Item Response Theory (IRT) is CAT traditional background. Bayesian networks (BNs) can offer IRT a powerful language for describing dependencies between skills and modeling richer tasks [1]. Although several researchers have explored BNs in educational assessment, real-world applications and extensive studies of their effectiveness are hardly found in the literature. In this work, we present a general procedure for BNs-based CAT and we test it in a real-world benchmark about German language proficiency assessment.

2 Adaptive Testing by Bayesian Networks

Students skills are modeled by a set $X := (X_1, X_2, \ldots, X_n)$ of categorical variables whose joint probability $P(X)$ is described by a BN through (i) a directed acyclic graph whose nodes represent the variables in $X$; (ii) conditional probability tables (CPTs) $P(X_i|\Pi_{X_i})$, $i = 1, \ldots, n$, where $\Pi_{X_i}$ is the joint variable of the parents (i.e., the immediate predecessors) of $X_i$ (see, e.g., Fig. 1 for the model used in the German language assessment). We point the reader to [2] for the theoretical notions about BNs. To evaluate the informativeness level about $X$ provided by $P$, we adopt the entropy $H(X) := -\sum_x P(x) \cdot \log P(x)$. Low entropy levels indicate high informativeness. To evaluate the student we formulate a number of questions, described as a collection of variables $Y := (Y_1, \ldots, Y_m)$. Each question node is represented as a leaf child of the background skills “required”
to answer it. To make our approach adaptive, we chose the \((k + 1)\)-th question to be asked based on the \(k\)-th previous answers \(y_1, \ldots, y_k\), by minimizing the conditional entropy \(H(X|y_1, \ldots, y_k, Y_{k+1}) := - \sum_{y_{k+1}} H(X|y_1, \ldots, y_k)P(y_{k+1})\).

Finally, we stop the test when the entropy \(H(X|y_1, y_2, \ldots, y_n)\) is sufficiently low.

An Application to Language Assessment. We use the answers of 170 students to 95 questions about German language to reproduce our CAT approach. The Traditional Evaluation Method (TEM) assigns to each skill a level A1, A2, B1, B2 by setting thresholds on the fraction of correct answers. We compare TEM with the independent skills model (IBN) and the tree (TBN) topology in Fig. 1.

Table 1 shows in the non-adaptive case the relative agreement between the TEM, IBN and TBN, and the internal consistency of the three tests evaluated using the split-half methodology. Both BN approaches have larger reliability than TEM. Concerning the adaptive case, Fig. 2 shows the relative agreement of the adaptive IBN and TBN with the non-adaptive TBN, and the average number of questions asked. Both models show a strong reduction in the number of questions as the entropy threshold increases. For instance, using the TBN model, we can save 20 questions on average at the price of only a 3\% accuracy reduction. This shows that a relevant number of question are little informative and could be avoided by means of an adaptive approach.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Relative agreement</th>
<th>Split-half reliability</th>
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</thead>
<tbody>
<tr>
<td>TEM/IBN</td>
<td>0.80</td>
<td>0.87</td>
</tr>
<tr>
<td>TEM/TBN</td>
<td>0.79</td>
<td>0.87</td>
</tr>
<tr>
<td>IBN/TBN</td>
<td>0.98</td>
<td>0.95</td>
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</tbody>
</table>

Table 1. Relative agreement between models and their split-half reliability.

References